On Intergranular Void Ratio of Loose Sand with Small Amount of Fines

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Abstract: Traditionally, void ratio has been used as an index to predict stress-strain behaviour of soil under steady state framework. Recent publications show that void ratio is not a proper index for gap graded sand with fines. Therefore, intergranular void ratio is proposed to resolve this problem. It is found that intergranular void ratio can predict stress-strain response of sand with low fines content (say 10%) only. To extend the applicability of intergranular void ratio to higher fines content, an additional parameter "*b*" has been introduced by some researchers in the calculation of intergranular void ratio. The parameter "*b*" represents the fraction of fines that actively take part on the parent sand force structure. However, there is a lack of theoretical basis for determining a *b*-value. Different "*b*" values have been selected by different researchers to enable the fitting of a single trend line, but this paper examines the physical significances of "*b*" by considering particles packing. This leads to a semi-empirical equation for predicting the value of "*b*" based on fines size and fines content. Published data from different researchers appears to be in support the proposed equation.

1 INTRODUCTION

Most of the previous laboratory studies on liquefaction were confined to clean sand although loose sand with fines is not uncommon. It has been understood, since the 1960's, that the presence of fines in some manner affects the resistance to liquefaction of sand. But systematic studies on sand with fines have been relatively limited. Some recent studies showed a decrease of liquefaction resistance up to a certain limiting fines content followed by an increase in liquefaction resistance (Altun *et al*. 2005; Thevanayagam 1998; Xenaki & Athanasopoulos 2003; Yang *et al*. 2006a). But difficulties arise to explain these behaviours under Steady State (or Critical State) frame work. A unique SS or Critical State line or curve cannot be obtained.

One of the possible reasons that lead to a lack of a unique SS line/curve may be due to the inactive participation of fines in the force structure of a sand-fines mix. To resolve this problem, a new state variable referred to as intergranular void ratio has been proposed as an alternative to void ratio. To further extend the applicability of intergranular void ratio, Thevanayagam (1999) introduced a parameter "*b*" which presents the fraction of fines that actively take part on the force structure. But the physical significance of "*b*" is a controversial topic in literature (Ni *et al*. 2004; Thevanayagam 2001). Different researchers selected different values of "*b*" so that data point points from different fines content are located within a narrow band.

The objective of this paper is to examine the physical basis of intergranular void ratio and the "*b*" parameter. A semi-empirical equation for predicting "*b*" is presented based on the binary packing concept. The applicability of the equation is verified for different types of sand with fines.

2 LITERATURE REVEIW

2.1 *Intergranular Void Ratio*

According to literature, Mitchell (1976) first used the concept of intergranular void ratio to determine the inactive clay content on soil structure. One year later Kenney (1977) found, soil containing clay minerals and water less than 40% to 50% of total volume showed a residual strength is equal to that of the granular mineral (quartz) only. Troncoso & Verdugo (1985) also found similar out come from cyclic triaxial experiment on tailing sand with 30% fines. Although these results support the concept of intergranular void ratio, Kuerbis *et al*. (1988) may be the first researcher that used intergranular void ratio as the basis for comparing undrained shear strength behaviour. He suggested that fines particles simply be occupying gaps in the sand skeleton and therefore, the measured behaviour is controlled by sand skeleton only. Thus, neglecting the volumes of fines he calculated intergranular void ratio as

$$e_{skeleton} = \frac{V_T G_S \rho_W - (M - M_{silt})}{(M - M_{silt})} \tag{1}$$

He observed almost similar soil strength behaviour at the same intergranular void ratio. Georgiannou *et al*. (1990) also found similar out come for clayey sand (Ham river sand) and proposed intergranular void ratio as

$$e_g = \frac{\text{Volume of voids} + \text{volume of clay}}{\text{volume of granular phase}} \tag{2}$$

He observed effective stress paths in UC tests were almost identical for similar intergranular void ratio. Thevanayagam (1998) found unique steady state line for a host sand with non-plastic fines by using intergranular void ratio as

$$e_s = \frac{e + f_c}{1 - f_c} \tag{3}$$

where, e = void ratio and f_c = fines content in decimal. Though different researcher used different equation for intergranular void ratio, Chu & Leong (2002) showed that the equations used for calculating intergranular void ratio are almost identical. However this definition of intergranular void ratio is not applicable for entire range of fines content. Fines particles come in between the contact of sand grain with increasing fines and soil act as a composite structure. The active contacts of fines on sand force structure are needed to be considered by a proper parameter. Considering the contribution of fines to the force structure, Thevanayagam (1999; 2000) redefined intergranular void ratio as

$$e_c = \frac{e + (1-b)f_c}{1-(1-b)f_c} \quad (4)$$

where b = fraction of fines which actively take part in the sand force structure. This equation further extends the applicability of intergranular void ratio. When b = 0, it represent the previous equation of intergranular void ratio and when b > 0, it also allow gradual contribution of fines in sand force structure. The successful application of "*b*" has been discussed in literature by Ni *et al.* (2004), Thevanayagam (2000) and Yang *et al*. (2006b).

However physical significance and magnitude of "*b*" is a controversial topic in literature. Some researchers assigned a constant value to "*b*" for a given sand fines mix. For example b = 0.35 for Ottawa sand-silt mix (Thevanayagam 2001), b =0.25 for Toyoura sand-silt mix (Ni *et al* 2004). Thevanayagam (2001) suggested that "*b*" depends on $C_{uc}C_{uf}^2/R_d$, where C_{uc} = uniformity coefficient of coarse material (sand), C_{uf} = uniformity coefficient of fine grain and $R_d = D_{50}/d_{50}$ (D = size of sand and d = size of fines). However, Ni *et al*. (2004) suggested that "*b*" depends upon D_{10}/d_{50}. They found b = 0.7 for Old Alluvium sand (Singapore) with non-plastic fines. They also analyzed data from Zlatovic & Ishihara (1995) and found that b = 0.25 can be used for fines content up to 30%. Recently Yang *et al*. (2006b) suggested that "*b*" is not a constant value for Hokksund sand with Chengbei non-plastic fines: b = 0.25 for up to threshold fines content and b = 0.40 at threshold fines content, where threshold fines content was taken as 30%. The above discussion highlighted that the value of "*b*" are selected rather than determined or predicted based on its physical meaning.

2.2 *Binary Packing Concept*

As the "*b*" parameter was supposedly introduced to reflect the contribution of fines to the force structure, binary packing studies were examined. To see the effect of size ratio and relative amount on binary particle packing, McGeary (1961) performed some experiments on spherical balls of different size and relative composition. He found, if large particles are sufficiently large then small particles are able to migrate and fit between the gaps of larger particles otherwise any increase in small particles will fall in between the contacts of large particles. He found that the limiting size (diameter) ratio of large and small particle is at least D/d = 7 to migrate small particles in between the gaps of large particles. This is consistent with geometric calculation of fitting a small sphere between large spheres without altering the contacts between large spheres as illustrated in Fig. 1. Lade (1998) analyze McGeary's (1961) data and showed that when D/d ≥ 7, small particle migrate and fits in between the gaps of large particle and produce the "lowest" e_{min} (minimum void ratio) of binary mix, and that rate of decreasing in e_{min} with D/d is negligible when D/d >> 7. This is illustrated in Fig. 1. When D/d < 7, the arrangement cannot reach the lowest e_{min} because small spheres will come in between the large spheres. His study also showed that increase in small particles reduced the void ratio until they fill the gaps of large particles and then small particles will come in between the contacts of large particles and increase the void ratio again as shown in Fig. 2. These findings indirectly highlight the factors that influence the "extent of contacts" between large and small particles of a binary mix.

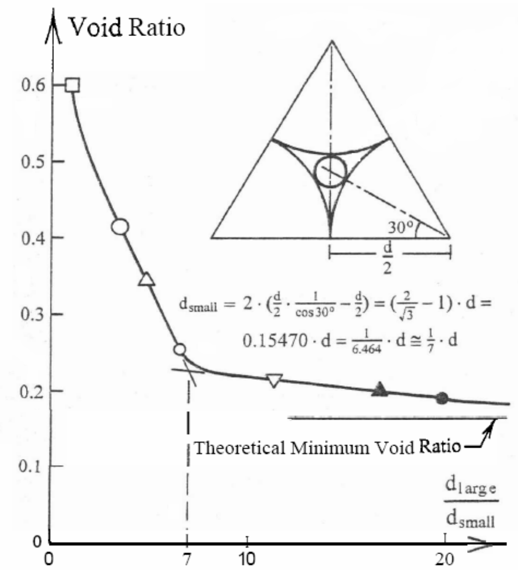


Fig. 1 Effect of diameter ratio on minimum void ratio of binary mix (After Lade *et al*. 1998)

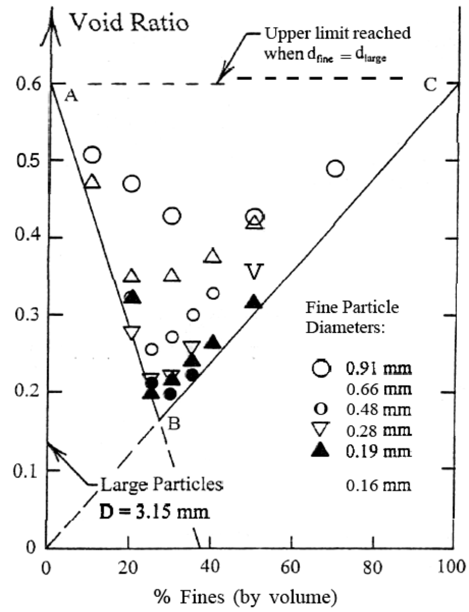


Fig. 2 Effect of fines content on minimum void ratio of binary mix (After Lade *et al*. 1998)

2.3 *Factors Affecting "b" from Binary Packing Concept*

The "*b*" value is in fact a measure of the extent of contacts between the large and small particles. Thus the above study on binary packing can be used to establish the factors that affect the *b*-value. For sand with fines, D can be considered to be the size of sand particles whereas d is the size of silt particles. The value of "*b*" depends on two variables; relative amount of small particles and size ratio, D/d. The size ratio D/d = 7 is a significant number that define the initial value and the rate increase of "*b*" with fines content. They are discussed below:

- *When D/d < 7*: "*b*" increased rapidly for any increment of fines content as small particles will fall in between the contacts of large particles from the very beginning and the rate of increment of “b” is increased as diameter ratio approach to unity, D/d → 1.

- *When D/d ≥ 7*: “b” is negligible up to certain fines content as fines migrate and fit in between the gaps of large particles until they fill it and then ‘b” increases gently with fines content as fines come in between the contacts of large particles
- *When D/d >>7*: “b” follows almost similar trend of D/d ≥ 7, because much smaller particles also shows similar behaviour of migration in between large particles.

3 PROPOSED FORMULA FOR “b”

Since sand and fines are not single size, we will instead use D_{10} for sand and d_{50} for fines. Therefore the size ratio is now D_{10}/d_{50}. An empirical equation for “b” is proposed depending on the basic characteristics from binary packing concept.

$$b = \left(1 - e^{-\left(nf_c^{\,n}/k\right)}\right)\left(r\frac{f_c}{f_{thre}}\right)^r \tag{5}$$

where $r = (D_{10}/d_{50})^{-1} = d_{50}/D_{10}$, f_{thre} is the threshold fines content and $k = (1- r^{\,0.25})$.

The first factor of Eq. (5) gives the overall trend of increase in b with fines content, FC. The parameter “n” governs the initial rate of increase, $\partial b/\partial f_c$. A rapid rate of increase is achieved with n = 1, whereas a “flat increase” is achieved when if n = 2. An average value n = 1.5 is used to cover wide range of sand with fines. The influence of size ratio on the value of the first factor is ensured through k. In general, a higher size ratio will give to a higher k value, which then leads to a lower value for the first factor. Further more, at lower size ratio of $D_{10}/d_{50} < 5$, the value of the first factor changes rapidly with size ratio, but at higher size ratio, say of $D_{10}/d_{50} > 7$, the value of the first factor changes very slowly. These trends are consistent with the requirements inferred by a binary packing consideration.

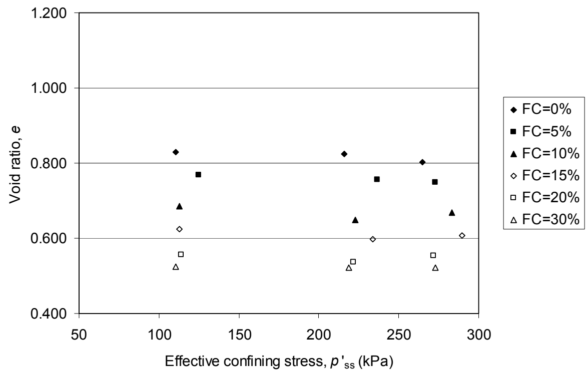
The second factor of the equation represents the rate of increment of “b” with fines content and size ratio. Lower values of r ensure gentle increment of “b” and higher values of r ensure rapid increment. This factor also ensures that $b \rightarrow 0$ when fines content approaches zero.

4 VERIFICATION OF THE EQUATION

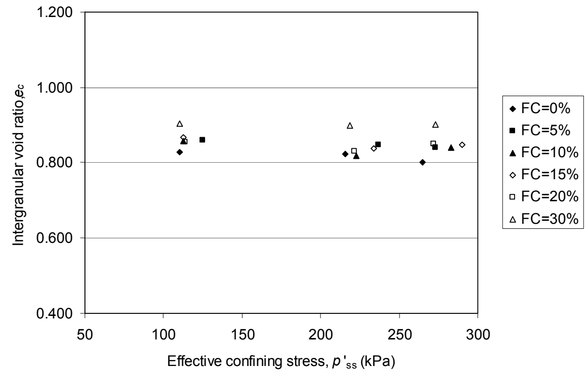
We used Eq. (5) to calculate a “b” value, which can then be substituted into Eq. (4) to yield an intergranular void ratio. We then examined whether a unique relationship can be obtained utilizing the intergranular void ratio as the alternative state parameter. Two types of behaviour are examined: the SS data points from monotonic undrained shearing and the cyclic mobility behaviour from cyclic triaxial testing. Both types of data correspond to large deformation behaviour with significant generation of pore water pressure.

4.1 *Steady State Behaviour from Monotonic Undrained Shearing*

Yang *et al*. (2006a) studied the influence of Chengbei non-plastic silt using Hokksund sand. Their SS data points in the e-p'_{ss} space are shown in Fig. 3(a), where p'_{ss} = mean effective stress and e = void ratio at Steady State. It is found that the data points gradually moved downward with increasing FC, the fines content in %. Their data points were re-plotted by using intergranular void ratio in lieu of void ratio in Fig. 3(b). All data points for different FC follow a single trend. Therefore, equation (5) successfully gives a unique SSL by using intergranular void ratio as the state variable.



(a)



(b)

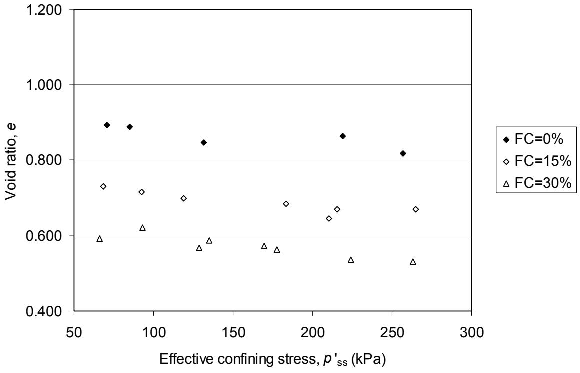
Fig. 3 SS data for Hokksund sand with Chengbei non-plastic fines; (a) after Yang *et al*. 2006a, (b) intergranular void ratio calculated using Eq. (5).

Huang *et al*. (2004) performed a series of laboratory tests on reconstitute sample of Mai Liao Sand (MLS) with silty fines from Central Western Taiwan. The data points manifest a significant and considerable spread in the e-p'_{ss} space as shown in Fig. 4(a). However if intergranular void ratio calculated using Eqs. (4) and (5) is used as the state parameter, all SS data points come to a narrow band as shown in Fig. 4(b). This again demonstrates the success of Eq. (5).

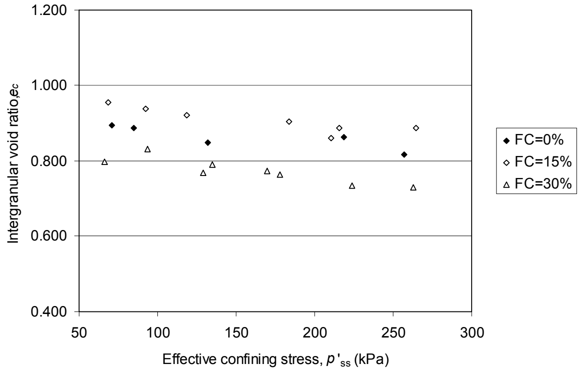
Ni *et al*. (2004) studied the influence of plastic and non-plastic fines on Old Alluvium sand from Singapore. Their data points in the e-p'_{ss} space shown as the hatched zone of Fig. 5 manifested significant spread. However, all SS data points follow essentially a unique curve if they are plotted using intergranular void ratio. It is interesting to note that Ni *et al*. (2004) assume b = 0.7 for calculating intergranular void ratio and also got an approximately unique curve for the SS data points.

Thevanayagam *et al*. (2002) presented SS data points for Foundary sand mixed with crushed silica (as fines). The data points in the e-p'_{ss} space manifest significant spread (Fig. 6(a); but an approximately unique SS curve was obtained if intergranular void ratio is used as the state variable as shown in Fig. 6(b).

Based on the data presented above, a unique SS relationship can be obtained in terms of the intergranular void ratio calculated using the "b" value predicted by Eq. (5).



(a)



(b)

Fig. 4 SS data for Mai Liao sand with fines; (a) after Huang *et al* 2004, (b) intergranular void ratio calculated using Eq. (5).

4.2 *Cyclic Mobility Behaviour*

Vaid (1994) performed cyclic triaxial tests on Brenda 20/200 sand with non-plastic fines. Brenda sand is angular tailing sand. He found cyclic stress ratio, CSR versus void ratio data points moved downward with the increment of fines as shown in Fig. 7(a). However, an essentially unique relationship is obtained if intergranular void ratio calculated using Eqs. (4) and (5) is used as the state variable as shown in Fig. 7(b).

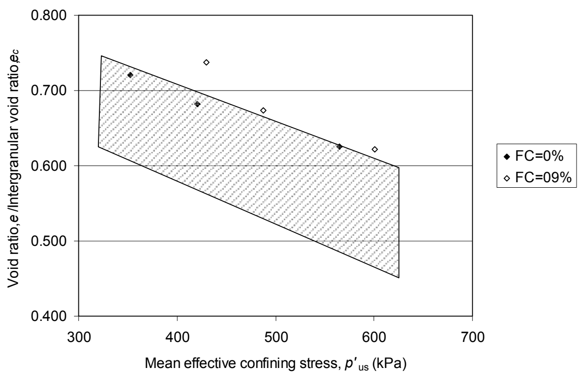
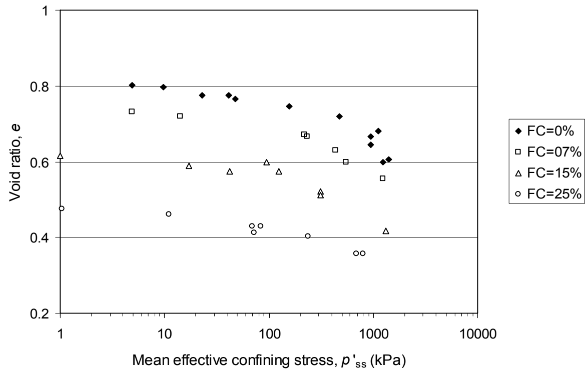
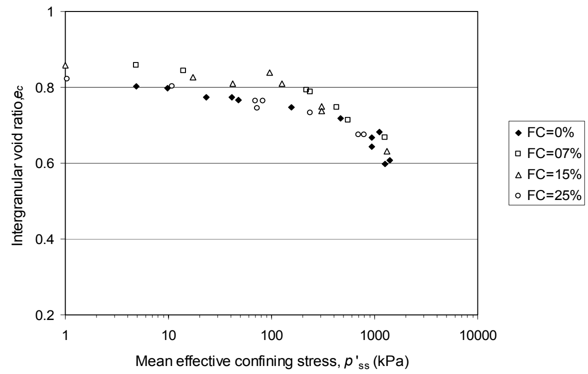


Fig. 5 SS data for Old Alluvium sand with fines

Polito (1999) preformed cyclic triaxial tests on Yatesville sand with Yatesville fines. Yatesville sand is a poorly graded, medium to fine sand which was obtained from a dam site in Kentucky. Fines was derived from the fine-grained portion of Yatesville silty sand. Moist temping method was used for sample preparation. As shown in Fig. 8(a), cyclic resistance data points moved down with the increase in FC. The data points were however brought closer when intergranular void ratio was used as the state variable in the plot as shown in Fig. 8(b).



(a)



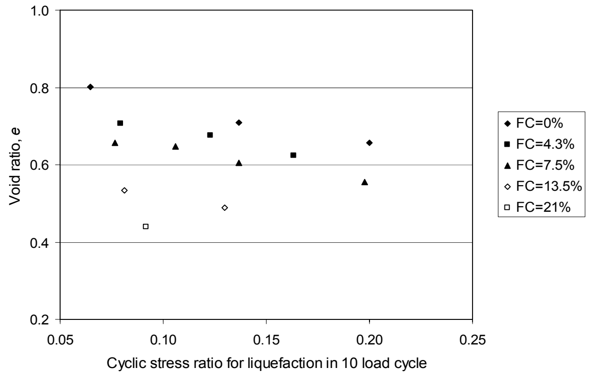
(b)

Fig. 6 SS data for Foundry sand with non-plastic fines; (a) after Thevanayagam *et al*. 2002, (b) intergranular void ratio calculated using Eq. (5).

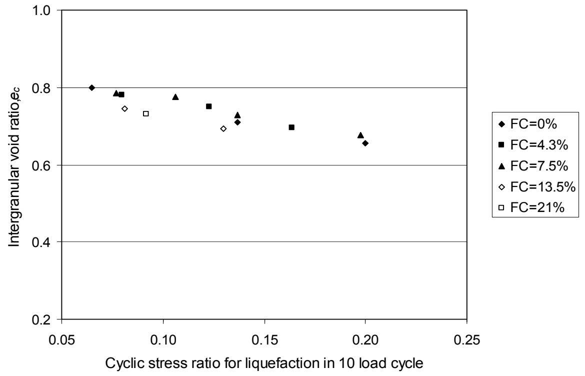
Polito & Martin (2001) preformed a series of cyclic triaxial tests on Monterey No. 0/30 sand with Yatesville fines. Fig. 9(a) showed a clear distinct trend of moving down cyclic resistance curve with increasing FC. A single narrow band of cyclic resistance curve is obtained when that data are plotted against intergranular void ratio as shown in Fig. 9(b).

Thevanayagam & Martin (2002) studied the effect of fines on liquefaction using Ottawa sand with non plastic silt. Moist temping and air deposition method was used for sample preparation. They used number of cycle required to achieve cyclic liquefaction at a cyclic stress ratio of 0.20 as the basis for comparison. Fig. 10(a) showed that the data points do not follow a unique curve but manifest a wide spread. However, if intergranular void ratio calculated using Eqs. (4) and (5) is used as the state variable, the data points are located within a narrow band as shown in Fig. 10(b).

Based on the above data analysis, Eq. (5) can be used to predict the b value for calculating the intergranular void ratio, which in turn can be uniquely related to some measure of cyclic mobility behaviour.

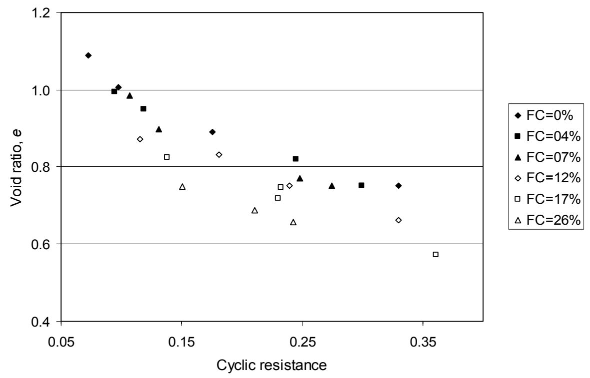


(a)

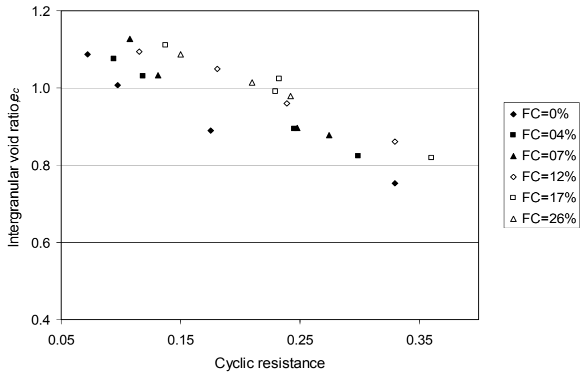


(b)

Fig. 7 Cyclic data for 20/200 Brenda sand with silty fines; (a) after Vaid 1994, (b) intergranular void ratio calculated using Eq. (5).

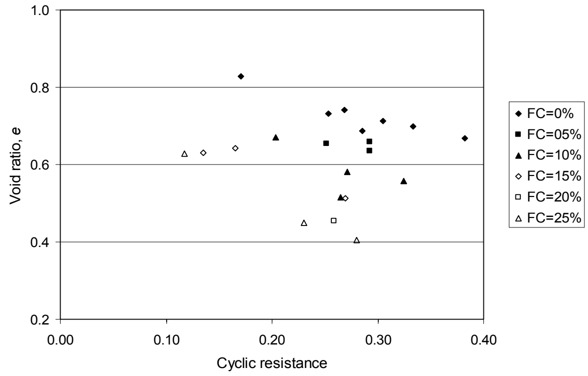


(a)

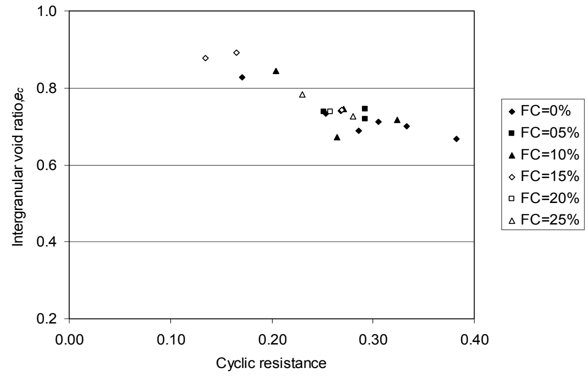


(b)

Fig. 8 Cyclic data for Yatesville sand and fines; (a) data after Polito 1999, (b) intergranular void ratio calculated using Eq. (5).

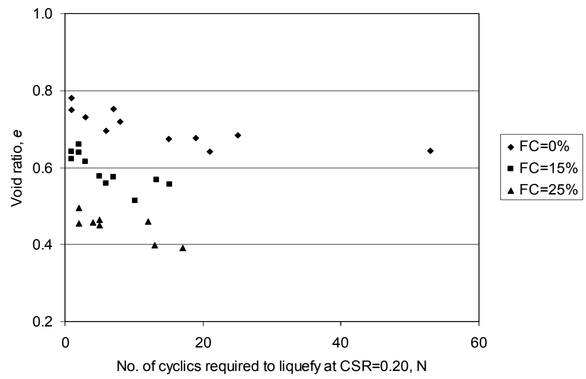


(a)

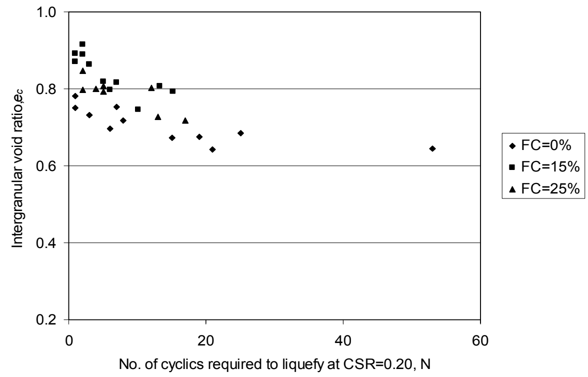


(b)

Fig. 9 Cyclic data for Monterey sand and Yatesville fines; (a) after Polito & Martin 2001, (b) data calculated using Eq. (5).



(a)



(b)

Fig. 10 Cyclic data for Ottawa sand with non-plastic fines;
(a) Thevanayagam & Martin 2002, (b) data calculated using Eq. (5).

5 CONCLUSION

The fraction of fines that actively take part on sand force structure is presented by the "*b*" parameter, which in turn, affects the intergranular void ratio. The parameter "b", however, is not a constant, but is dependent on a number of factors.

As the size of fines is in general significantly smaller than that of sand, the binary packing concept can be used to establish

- The size ratio and fines content (as a fraction of the threshold fines content) are the important factors that influence the value of "b".
- The constraints on the overall trend for the variation in "b" with the above factors.

Based on the above, a semi-empirical relationship was proposed for the predicting the "b" value.

The validity of the proposed equation was tested against published data that covers eight different sources from different regions of the world. The "b" values so predicted yield, irrespective of fines content, either a unique SS curve based on intergranular void ratio (ie, in the e_c versus p_{SS} space) or a unique correlation between intergranular void ratio and cyclic mobility response. Therefore, the proposed equation may be argued to be valid for a wide range of sand fines mixture. However, it is noted that the data base is essentially for non-plastic fines and one may not be extrapolated the findings to plastic fines.

REFERENCES

Altun, S., Goktepe, A. B. & Akguner, C. (2005) "Cyclic Shear Strength of Silts and Sands under Cyclic Loading." Earthquake Engineering and Soil Dynamics (GSP 133), 158, 33.

Chu, J. & Leong, W. K. (2002). "Effect of fines on instability behaviour of loose sand." Geotechnique, 52(10), 751-755.

Georgiannou, V. N., Burland, J. B. & Hight, D. W. (1990). "The undrained behavoiur of clayey sands in triaxial compression and extension." Géotechnique, 40(3), 431-449.

Huang, Y.-T., Huang, A.-B., Kuo, Y.-C. & Tsai, M.-D. (2004). "A laboratory study on the undrained strenght of silty sand from Central Western Taiwan." Soil Dynamics and Earthquake Engineering, 24, 733-743.

Kenney, T. C. (1977) "Residual strength of mineral mixture." Proc. 9th International Conference of Soil Mechanics and Foundation Engineering, 1, 155-160.

Kuerbis, R., Negussey, D. & Vaid, Y. P. (1988) "Effect of Gradation and Fine Content on the Undrained Response of Sand." In Hydraulic fill structure,Geotechnical Special Publication 21, ASCE, New York, 330-345.

Lade, P. V., Liggio, C. D. & Yamamuro, J. A. (1998). "Effects of non-plastic fines on minimum and maximum void ratios of sand." Geotechnical Testing Journal, 21(4), 336-347.

Mcgeary, R. K. (1961). "Mechanical Packing of Spherical Particles." Journal of the American Ceramic Society, 44(10), 513-522.

Mitchell, J. K. (1976). Fundamental of soil behaviour, John Wiley & Sons, Inc.

Ni, Q., Tan, T. S., Dasari, G. R. & Hight, D. W. (2004). "Contribution of fines to the compressive strength of mixed soils." Geotechnique, 54(9), 561-569.

Polito, C. P. (1999). "The effects of non-plastic and plastic fines on the liquefaction of sandy soils," PhD thesis in Civil Engineering.

Polito, C. P. & II, J. R. M. (2001). "Effects of nonplastic fines on the liquefaction resistance of solids." Journal of Geotechnical and Geoenvironmental Engineering, 127(5), 408-415.

Thevanayagam, S. (1998). "Effect of Fines and Confining Stress on Undrained Shear Strength of Silty Sands." Journal of Geotechnical and Geoenvironmental Engineering, 124(6), 479-491.

Thevanayagam, S. (1999). "Effect of Fines and Confining Stress on the Undrained Shear Strength of Silty Sands - Closure." Journal of Geotechnical and Geoenvironmental Engineering, 1024-1027.

Thevanayagam, S. (2000) "Liquefaction potential and undrained fragility of silty soils." Proc. 12th World Conf. Earthquake Egnineering, Aucland, New Zealand, Paper #2383.

Thevanayagam, S. (2001). "Role of intergranular contacts on the mechanisms causing liquefaction and slope failures in silty sands" Annual project summary report, USGS Award Number: 01HQGR0032. US Geological Survey, Department of Interior,USA.
http://erpweb.er.usgs.gov/reports/annsum/vol43/pt/g0032.pdf

Thevanayagam, S. & Martin, G. R. (2002). "Liquefaction in silty soils-screening and remediation issues." Soil Dynamics and Earthquake Engineering, 22(9-12), 1035-1042.

Thevanayagam, S., Shenthan, T., Mohan, S. & Liang, J. (2002). "Undrained fragility of clean sands, silty sands, and sandy silts." Journal of Geotechnical and Geoenvironmental Engineering, 128(10), 849-859.

Troncoso, J. H. & Verdugo, R. (1985) "Silt content and Dynamic behaviour of tailing sands." Proc. 11th International Conference on Soil Mechanics and Foundation Engineering, 3, 1311-1314.

Vaid, Y. P. (1994). ""Liquefaction of silty soils" in Ground Failure under Seismic Conditions." Geotech. Spl. publ. No. 44, S. Prakash and P. Dakoulas (eds.), 1-16.

Xenaki, V. C. & Athanasopoulos, G. A. (2003). "Liquefaction resistance of sand–silt mixtures: an experimental investigation of the effect of fines." Soil Dynamics and Earthquake Engineering, 23, 183-194.

Yang, S. L., Lacasse, S. & Sandven, R. F. (2006a). "Determination of the transitional fines content of mixtures of sand and non-plastic fines." Geotechnical Testing Journal, 29(2), 102-107.

Yang, S. L., Sandven, R. & Grande, L. (2006b). "Instability of sand-silt mixtures." Soil Dynamics and Earthquake Engineering, 11th International Conference on Soil Dynamics and Earthquake Engineering (ICSDEE): Part II, 26(2-4), 183-190.

Zlatovic, S. & Ishihara, K. (1995) "On the influence of nonplastic fines on residul strength." Proceedings of IS-TOKY0'95/ The First International Conference on Earthquake Geotechnical Engineering/Tokyo/ 14-16 November 1995, Tokyo, Japan, 1, 239-244.